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A STUDY ON ARTINIAN RINGS

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Let A be a left Artinian ring, and N the radical of A. The famous theorem of Hopkins [5] states that A is left Noetherian if A contains a left or right identity. Recently, Xŭ [14] obtained a theorem, which was noticed by the first author of this paper to be equivalent to saying as follows [13] : A is left Noetherian if and only if the trivial left A-module N/AN is finite. The second author, then, reproved these theorems together by further investigation of the module N/AN [10]. Incidentally, directly after the preparation of [10], Levy [8] showed that there is a lot of indecomposable left Artinian rings which are not left Noetherian and so contain no left or right identity. Therefore it is an interesting subject to study left Artinian rings without the assumption of such an identity.

Thus, in the previous paper [11], we considered a left Artinian ring A and its powers A^k assuming that A contains no left or right identity. We explicitly determined the radical of A^k, and proved that every A^k can not contain a left or right identity under a certain condition. Further it was proved that A^k is left Noetherian for all $k \geq 2$, no matter whether A is left Noetherian or not.

However, there are many theorems in [10] and [11] which could be obtained under more general situation. For example, Theorem 2 of [11] could be proved only by the condition that N is nilpotent and A/N has an identity (Theorem 3). One of the purposes of the present paper is to remark on these points and to restate some theorems there so generally as to include many other related theorems in [1], [3], [7], [10], [11] and [15]. The other one is to develop our study further and to reprove a theorem of Szász [12, Satz 4] together with theorems of Huynh recently obtained in [3]. Although Szász's theorem has been carried over to rings with minimum condition only for principal right ideals [2] (in fact, as an easy combination of [2, Hilfssätze 1, 2] and [4, Theorem 1]), our proof gives a concrete description of the Szász's decomposition.

According to our first purpose, we will begin with as general a consideration as possible. We let A be a non-zero ring possibly without identity, and $N = N(A)$ the *prime radical* of A. In general, for a subset B of A, the right (resp. left) annihilator of B in A will be denoted by $r(B)$ (resp. $l(B)$). In particular, if B consists of one idempotent e, we set $R_e = r(e)$, $L_e = l(e)$ and $T_e = r(e) \cap l(e)$.

1. In general, if e is an idempotent of A then $A = eA \oplus R_e = eAe \oplus R_e e \oplus eL_e \oplus T_e = Ae \oplus L_e$ (Peirce decompositions) and $R_e T_e = T_e^2 = T_e L_e$. For a moment, we assume that N is nilpotent and $\bar{A} = A/N$ has a left identity. Then the left identity of $\bar{A}$ can be lifted to an idempotent e of A. (If $A = N$, then 0 is considered as e.) Noting that $A = eA + N$, one will easily see that $N = eN \oplus R_e$. It is also easy to see the following for any positive integer k:

$$N^k = eN^k \oplus R_e N^{k-1}, \tag{1}$$

$$AN^k = eN^k \oplus R_e N^k, \tag{2}$$

$$A^k = AeA + T_e^k. \tag{3}$$

From (1) and (2), we readily obtain an (additive) group isomorphism

$$N^k/AN^k \cong R_e N^{k-1}/R_e N^k. \tag{4}$$

Theorem 1. *If N is nilpotent and A/N has a left identity, then the following are equivalent:*

1) *A has a left identity.*
2) *For each $a \in A$ there exists some $a' \in A$ with $a'a = a$.*
3) *$AN = N$.*

Proof. Obviously, 1) $\Longrightarrow$ 2) $\Longrightarrow$ 3). If $AN = N$ then $eN \oplus R_e = (eA \oplus R_e)N = eN \oplus R_e N$, whence it follows from the nilpotency of N that $R_e = R_e N = 0$. Thus, 3) implies 1).

By a theorem of Kuroš, if an additive Abelian group is Artinian then it is a direct sum of a finite number of quasi-cyclic and/or cyclic p-groups, and therefore a torsion group. A ring is said to be *strongly Artinian* if its additive group is Artinian. If ${}_AN$ is Artinian, it can be proved by a classical argument that N is nilpotent. The next includes [10, Propositions 8 and 9], [1, Satz], [12, Satz 3] and [7, Folgerung 2].

Theorem 2. *If ${}_AN$ is Artinian and A/N has a left identity, then R_e is a strongly Artinian right subideal of N. In particular, if A is left Artinian and torsion-free then A contains a left identity, and if A is left Artinian and nilpotent then A is a torsion ring.*

Proof. If $N^n = 0$, then R_e/R_eN, R_eN/R_eN^2, $\cdots$, $R_eN^{n-1}/R_eN^n = R_eN^{n-1}$ are Artinian groups by (4). Hence, R_e is obviously strongly Artinian.

Corollary 1. *Assume that ${}_AN$ is Artinian and A/N has a left* *identity. If A is an algebra over an infinite field K then A has a left identity.*

Proof. By Theorem 2, R_e is a strongly Artinian right subideal of N. Hence, if K is of characteristic 0, then $R_e = 0$. Meanwhile, if the infinite field K is of non-zero characteristic and $R_e \neq 0$, then R_e is infinite dimensional over the prime field of K. It is a contradiction. Thus we obtain $A = eA$ in either case.

In the rest of this section, we assume always that N is nilpotent and $\bar{A} = A/N$ has an identity, and let n be the nilpotency index of N. Let e be an idempotent lifted from the identity of $\bar{A}$, and set $R = R_e$, $L = L_e$ and $T = T_e$. Then $N = Ne \oplus L = eNe \oplus Re \oplus T = eN \oplus R$. We shall show that the prime radical N_k of A^k is $A^{k-1}N + NA^{k-1}$. More precisely we shall prove the following

Theorem 3. (a) $A^k = A^n + T^k = AeA + T^k = eAe \oplus Re \oplus eL \oplus (ReL + T^k)$.
(b) $N_k = A^{k-1}N + NA^{k-1} = A^k \cap N = eNe \oplus Re \oplus eL \oplus (ReL + T^k)$.
(c) $A^k/N_k \cong A/N$ *(ring-isomorphism)*.
(d) $A^k \cap T = ReL + T^k$.

Proof. (a) The proof is quite similar to that of [11, Theorem 2 (i)].

(b)—(d) As in the proof of [11, Theorem 2 (ii)—(iv)], we have $A^{k-1}N + NA^{k-1} = eNe + eL + Re + (ReL + T^k)$. Hence $A^k = eAe + (A^{k-1}N + NA^{k-1})$ and $eAe \cap (A^{k-1}N + NA^{k-1}) = eNe$. Since $A^{k-1}N + NA^{k-1}$ is a nilpotent ideal of A and $A^k/(A^{k-1}N + NA^{k-1}) \cong eAe/eNe \cong A/N$, our assertions are evident in whole.

Since N is nilpotent, Theorem 3 and (3) will give at once the next

Corollary 2. *The following are equivalent:*
1) $A^k = A^{k+1}$.
2) $N_k = N_{k+1}$.
3) $ReL \supseteq T^k$.
4) $AeA \supseteq N_k$.

Especially, $A = A^2$ *if and only if* $RL = T$, *and* $A^2 = A^3$ *if and only if* $RL = ReL$.

Corollary 3. *The following are equivalent:*
1) A^k *has a left (resp. right) identity.*
2) $A^k = A^{k+1}$ *and* $A^kN \supseteq NA^k$ *(resp.* $NA^k \supseteq A^kN$*)*.

3) $A^k \cap N = A^k N$ (*resp.* $A^k \cap N = NA^k$).

4) $A^k N_k = N_k$ (*resp.* $N_k A^k = N_k$).

Proof. 1) $\Longrightarrow$ 2). Since $A^k = A^{k+1}$, by Theorem 3 (b) we have $N_k = N_{k+1} = A^k N + NA^k = A^k(A^k N + NA^k) = A^k N$.

2) $\Longrightarrow$ 3). By Theorem 3 (b), $A^k \cap N = N_k = N_{k+1} = A^k N + NA^k = A^k N$.

3) $\Longrightarrow$ 4). From the above equality, $A^{k-1}N + NA^{k-1} = A^k N$. Since $A^{k-1} N \supseteq A^k N$, we have then $A^{k-1}N = A^k N$. Hence $A^k N_k = A^{2k} N = A^k N = N_k$.

4) $\Longrightarrow$ 1). By Theorem 3 (c), A^k possesses the same property as A in the assumption of Theorem 1. Hence A^k has a left identity by 3) of Theorem 1.

The following examples illustrate Corollary 3.

Examples. (1) Let $A = \begin{pmatrix} \boldsymbol{Q} & \boldsymbol{Q} & \boldsymbol{Q} \\ 0 & 0 & \boldsymbol{Q} \\ 0 & 0 & 0 \end{pmatrix}$. Then A is a semi-primary ring (N is nilpotent and A/N is left Artinian) and subdirectly irredudible (and hence indecomposable). Obviously, $A \neq A^2 = A^3$, $A^2 N_2 = N_2$ and e_{11} is a left identity of A^2.

(2) Let $A = \begin{pmatrix} \boldsymbol{Q} & 0 & \boldsymbol{Q} & 0 \\ 0 & 0 & \boldsymbol{Q} & \boldsymbol{Q} \\ 0 & 0 & 0 & \boldsymbol{Q} \end{pmatrix}$, where the addition is as usual and the multiplication is defined by

$$\begin{pmatrix} x_{11} & 0 & x_{13} & 0 \\ 0 & 0 & x_{23} & x_{24} \\ 0 & 0 & 0 & x_{34} \end{pmatrix} \begin{pmatrix} y_{11} & 0 & y_{13} & 0 \\ 0 & 0 & y_{23} & y_{24} \\ 0 & 0 & 0 & y_{34} \end{pmatrix} = \begin{pmatrix} x_{11}y_{11} & 0 & x_{11}y_{13} & 0 \\ 0 & 0 & 0 & x_{23}y_{34}+x_{24}y_{11} \\ 0 & 0 & 0 & x_{34}y_{11} \end{pmatrix}.$$

Then A is a ring which is semi-primary and indecomposable. As can be easily seen, $A \neq A^2 = A^3$, $A^2 N_2 \neq N_2$ and A^2 has no left identity. (The multiplication is associative, because A is essentially a quasi-matrix ring over $\boldsymbol{Q}$ [9].)

2. As is well known, every left Artinian ring is semi-primary. The prime radical N of a left Noetherian ring A is nilpotent by Levitzki's theorem (see, e.g. [6, Theorem VIII. 4. 1); further, as can be easily seen, A/N is left Artinian if and only if N is the intersection of a finite number of maximal left ideals. In case A is semi-primary, we consider an idempotent e lifted from the identity of $\bar{A} = A/N$, and set $R = R_e$, $L = L_e$ and $T = T_e$. We begin this section with the following lemma which includes [5, 6. 6] and [11, Theorem 6].

Lemma 1. *If A is left* (*resp. right*) *Artinian, then ReA* (*resp. AeL*)

is finite; especially Re (resp. eL) and ReL are finite.

Proof. Generally, if an additive Abelian group is Artinian and of bounded order, then it is finite. Let $e = e_1 + \cdots + e_t$ with pairwise orthogonal primitive idempotents e_i, where Re_i is non-zero or zero according as $i \leq k$ or $i > k$. Then $ReA = R(e_1 + \cdots + e_k)A$ is included in the strongly Artinian right ideal R (Theorem 2). It suffices therefore to prove that each e_i $(i \leq k)$ is of finite order. Now, let u be a non-zero element of Re_i whose order is, say m. If me_i is not contained in N then we can find an element v in e_iAe_i such that $(me_i)v = e_i$. But then $u = ue_i = u(me_i)v = muv = 0$, contrary to assumption. Hence, me_i is in N, and therefore e_i is of finite order.

In case A is left Artinian, we shall denote by D the maximal divisible, torsion subgroup of A. It is well known that $DA = AD = 0$ and $T = G \oplus D$ with a finite group G (see, e.g. [8]). Then, by Theorem 3 we have

$$A^k = eAe \oplus Re \oplus eL \oplus (ReL + G^k) \qquad (k \geq 2). \tag{5}$$

Now, we are at the position to state the following which includes the main theorems of [10], [11, Theorem 8] and [15, Theorems 3 and 4].

Theorem 4. (a) *If A is left Artinian then the following are equivalent:*

1) *A is left Noetherian.*
2) *$mN \subseteq AN$ for some positive integer m.*
3) *N/AN is finite.*
4) *R is finite.*
5) *T is finite.*
6) $D = 0$.

(b) *If A is left Noetherian then the following are equivalent:*

1) *A is left Artinian.*
2) *N is the intersection of a finite number of maximal left ideals and $mN \subseteq AN$ for some positive integer m.*
3) *N is the intersection of a finite number of maximal left ideals and N/AN is finite.*
4) *$\bar{A}$ is left Artinian and R is finite.*

(c) *If A is left Artinian then A^k is left Noetherian for all $k \geq 2$.*

Proof. (a) Obviously, 5) and 6) are equivalent. Since $R = Re + T$ and Re is finite by Lemma 1, 4) and 5) are equivalent. Next, by (4) the group RN^{k-1}/RN^k is isomorphic to N^k/AN^k. Hence, 1)$\Longrightarrow$2)$\Longleftrightarrow$3)$\Longleftrightarrow$4).

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Finally, we shall prove that 2) implies 1). We observe the following chain:

$$A \supseteq N \supseteq AN \supseteq N^2 \supseteq \cdots \supseteq AN^{n-1} \supseteq N^n = 0 .$$

Evidently, every trivial left A-module N^k/AN^k of bounded order is finite and each completely reducible left $\bar{A}$-module AN^k/N^{k+1} is of finite length. Hence, ${}_AA$ has a composition series.

(b) Obviously, $1) \Longrightarrow 3) \Longrightarrow 2)$. As in (a), by (4) we readily see that 4) implies 3). Finally, assume 2). Then, as was noted at the opening of this section, $\bar{A}$ is left Artinian. Observe again the chain $A \supseteq N \supseteq AN \supseteq N^2 \supseteq \cdots \supseteq AN^{n-1} \supseteq N^n = 0$. Evidently, each finitely generated, trivial left A-module N^k/AN^k of bounded order is finite and each Noetherian completely reducible left $\bar{A}$-module AN^k/N^{k+1} is of finite length. Hence, A is left Artinian, proving 1) and 4) (see (a)).

(c) Evidently, G^k is finite. Since $A^k \cap T = ReL + G^k$ by (5) and ReL is finite by Lemma 1, $A^k \cap T$ itself is finite. Note that A^k is left Artinian (see [5, p. 722] or [11, Theorem 1]) and that e is a principal idempotent of A^k. Then it is clear that A^k is left Noetherian by (a).

Corollary 4. *If A is left Artinian, then A has a unique maximal (left and right) Artinian ideal A_0 and a unique maximal strongly Artinian ideal $\sigma(A)$, which includes all strongly Artinian left ideals of A.*

Proof. The maximal divisible, torsion subgroup D of A is clearly a strongly Artinian ideal of A, and A/D is left Noetherian by Theorem 4 (a). It assures us of the unique existence of $\sigma(A)$.

3. In this section, we restrict our attention to left (and/or right) Artinian rings, and reprove a theorem of Szász [12, Satz 4] and theorems of Huynh [3, Sätze 2. 1, 2. 2]. We continue to use the conventions employed in the preceding section.

Let f be an idempotent of A. If a left A-module ${}_AM$ is Artinian then so is ${}_{fAf}fM$. Especally, if A is a left Artinian ring then so is fAf. The statements in the next lemma can be easily seen.

Lemma 2. *Let f be an idempotent of a left Artinian ring A.*

(a) *fAf is finite if and only if $\bar{f}\bar{A}\bar{f}$ is finite.*

(b) *If ${}_AM$ is Artinian and fAf is finite, then fM is finite.*

(c) *Let M be a left (resp. right) A-module. If f is primitive and fAf is infinite, then fM (resp. Mf) is either 0 or infinite.*

In what follows, we assume that A is a left Artinian ring, and set $e = e_1 + \cdots + e_t$ with pairwise orthogonal primitive idempotents e_i. Here, we index e_i as follows: For $i \leq r$, e_i is of infinite order; for $r < i \leq s$, e_i is of finite order but e_iAe_i is infinite; for $i > s$, e_iAe_i is finite. We set

$$e' = e_1 + \cdots + e_r,\ e'' = e_{r+1} + \cdots + e_t,$$
$$e^\# = e_{r+1} + \cdots + e_s,\ e^* = e' + e^\#,\ e^{**} = e'' - e^\#.$$

Lemma 3. *Let A be a left Artinian ring.*

(a) *Ae_i and e_iA are torsion-free for $i \leq r$; necessarily $e_jAe_i = e_iAe_j = 0$ for $i \leq r$ and $j > r$.*

(b) *$Re_i = 0$ for $i \leq s$.*

(c) *$e_jAe_i = 0$ for $i \leq s$ and $j > s$.*

Proof. (a) Let u be a non-zero element of Ae_i, and suppose that $u \in N^{m-1}e_i$ but $u \notin N^m e_i$. Then $M = N^{m-1}e_i/N^m e_i$ can be regarded as a right $\bar{e}_i\bar{A}\bar{e}_i$-module. Assume $ku = 0$ for a positive integer k. Setting $\bar{u} = u + N^m e_i \in M$, we have $\bar{0} = k\bar{u} = k(\bar{u}\bar{e}_i) = \bar{u}(k\bar{e}_i)$. Since clearly $k\bar{e}_i \neq \bar{0}$, there exists an element $\bar{a}$ in $\bar{e}_i\bar{A}\bar{e}_i$ such that $(k\bar{e}_i)\bar{a} = \bar{e}_i$. Hence, $\bar{0} = \bar{u}\bar{e}_i = \bar{u}$; it implies $u \in N^m e_i$, contrary to hypothesis.

(b) If not, there exists a positive integer m such that $RN^m e_i = 0$ but $M = RN^{m-1}e_i \neq 0$. By Lemma 2 (c), M is infinite. On the other hand, M is finite as a subset of the finite set $Re_i = Ree_i$ (Lemma 1). This is a contradiction.

(c) Suppose $e_jAe_i \neq 0$. Then, by Lemma 2 (b), e_jAe_i is finite. But this is impossible by Lemma 2 (c).

Now, by means of the above lemmas, we shall reprove [12, Satz 4] and [3, Sätze 2. 1, 2. 2].

Theorem 5. *Let A be a left Artinian ring, and $\tau(A)$ the largest torsion ideal of A.*

(a) *There exists uniquely an ideal A' such that $A = A' \oplus \tau(A)$.*

(b) *A' is either 0 or a left Artinian ring with left identity whose factor ring modulo $N(A')$ is the direct sum of a finite number of simple rings of characteristic 0.*

Proof. By Lemma 3 (a) and (b), we have

$$\begin{aligned} A &= eAe \oplus Re \oplus eL \oplus T \\ &= (e'Ae' \oplus e'L) \oplus (e''Ae'' \oplus Re'' \oplus e''L \oplus T). \end{aligned}$$

We can easily see that $A' = e'Ae' + e'L$ and $T'' = e''Ae'' + Re'' + e''L + T$

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are subrings of A. By Lemma 3 (a) and Theorem 2, A' is torsion-free and T'' is a torsion ideal such that $A'T'' = T''A' = 0$. Hence, T'' coincides with $\tau(A)$.

Theorem 6. *Let A be a (left and right) Artinian ring, and $\sigma(A)$ the maximal strongly Artinian ideal of A.*

(a) *There exists uniquely an ideal A^* such that $A = A^* \oplus \sigma(A)$.*

(b) *A^* is either 0 or an Artinian ring with identity whose factor ring modulo $N(A^*)$ is the direct sum of a finite number of infinite simple rings.*

(c) *If B is an Artinian ideal of A then $\sigma(B) = \sigma(A) \cap B$ and $B^* = A^* \cap B$.*

Proof. By Lemma 3 (b), (c) and the left-right symmetry of them, we can write

$$\begin{aligned} A &= eAe \oplus Re \oplus eL \oplus T \\ &= e^*Ae^* \oplus (e^{**}Ae^{**} \oplus Re^{**} \oplus e^{**}L \oplus T). \end{aligned}$$

As can be easily seen, e^*Ae^* and $S = e^{**}Ae^{**} + Re^{**} + e^{**}L + T$ are ideals of A. Hence, we obtain the ring-theoretic decomposition $A = e^*Ae^* \oplus S$. By Lemma 2 (b) and Theorem 2, S is strongly Artinian. Moreover, by Lemma 2 (c) every non-zero one-sided ideal of e^*Ae^* is infinite and contains no divisible, torsion subgroup. Recalling the theorem of Kuroš mentioned before Theorem 2, we can readily see that e^*Ae^* contains no strongly Artinian ideal and hence $S = \sigma(A)$. Now, remaining assertions will be immediate.

Corollary 5. *If A is an Artinian ring, then there exists uniquely an ideal $A^\sharp$ such that $A = A' \oplus A^\sharp \oplus \sigma(A)$. $A^\sharp$ is either 0 or an Artinian ring with identity whose factor ring modulo $N(A^\sharp)$ is the direct sum of a finite number of infinite simple rings of non-zero characteristic.*

Proof. It is enough to notice that $\tau(e^*Ae^*) = e^\sharp Ae^\sharp$ and $(e^*Ae^*)' = A'$.

Corollary 6. *Let A be an Artinian ring. If A is indecomposable and not left Noetherian, then A/D is finite.*

Remark. Let A be a left Artinian ring, and A_0 the unique maximal Artinian ideal of A (Corollary 4). By Theorem 6, there holds $A_0 = A_0^* \oplus \sigma(A_0)$. It is easy to see that A_0^* is an ideal of A and $\sigma(A_0)$ coincides with $\sigma(A)$. From these, [3, Satz 2. 3] will be evident.

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